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EXTREMAL METHODS IN LOGISTICS RESEARCH: A DEVELOPMENTAL SURVEY

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EXTREMAL METHODS IN LOGISTICS RESEARCH: A DEVELOPMENTAL SURVEY

by

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13. ABSTRACT

The problems of spatial and time-dependent analysis in logistics stimulated the creation and development of the wide classes of techniques conveniently summarized as "extremal methods," e.g. linear programming, semi-infinite programming, game theory, functional equations, geometric programming, chance-constrained programming, integer programming, fractional programming, interval programming, etc. A historical review of significant papers, their impact on logistical and mathematical method, and a survey of the current status of extremal methods is herein presented.

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Abstract

The problems of spatial and time-dependent analysis in logistics stimulated the creation and development of the wide classes of techniques conveniently summarized as "extremal methods," e.g. linear programming, semi-infinite programming, game theory, functional equations, geometric programming, chance-constrained programming, integer programming, fractional programming, interval programming, etc. A historical review of significant papers, their impact on logistical and mathematical method, and a survey of the current status of extremal methods is herein presented.

EXTREMAL METHODS IN LOGISTICS RESEARCH: A Developmental Survey

1. Introduction

The inventory which follows this introduction represents a compilation from the articles published in the Naval Research Logistics Quarterly. The list is by no means exhaustive, even at best, but it at least represents a convenient source for examining some of the uses of extremal methods in logistics and it portrays research aspects and developments in a relatively prominent form.

At the outset we should indicate that "extremization" or an "extremal method" refers to the optimizing operators such as "max" or "min" or "min max" or "max min," etc. It also extends to the operations such as "inf" or "sup" where a direction of optimization is indicated. Finally an "extremal equation" refers to any equation whose elements are required to satisfy prescribed extremization conditions or operations. This is by analogy, or extension, of the way one characterizes differential equations or integral equations in terms of derivative or integration operations, and so on.

2. Strategies for Applying Extremal Methods to Logistics Problems

A check of the articles referenced later in this paper will make it apparent that there has been a vigorous and "varied" growth in extremal methods. Ideally we would have liked to match these methods against significant classes of problems to portray their potential (or actual) uses and shortcomings. At a minimum we would have liked to arrange a two-way classification or matrix array in which we could have at least indicated where uses of linear programming, game theory or other such developments embodying extremal methods or approaches had been used on classes of logistics problems such as manpower planning or fleet provisioning with further breakdowns into recruitment and procurement, storage and transfer, etc. The format of the articles and the arrangements in the journal, however, prevented us from doing this. We were, however, able to make a rough classification of these articles by methods and problems addressed. To accompany this and provide perspective we can perhaps draw upon strategies of the kinds we have ourselves used in bringing extremal methods to bear on logistics problems.

This we might do in a somewhat informally structured way as follows:

A. Rationales for the Use of Extremal Methods
<u>Comment</u>. We have become so accustomed to discussions about the relevance or realism of particular objectives of optimization in game theory, economic theory, etc., that we can fail to remember that optimizations can also be used as artifacts to simplify an otherwise complex analysis or diminish ambiguity, and so on. We have discussed these uses elsewhere in considerable detail[1] and hence we simply indicate some of the related further considerations as follows:

1. Representation--Ease in Formulating Problems and Assessing Potential Further Consequences. (Example: Game Theory as used in representing and studying traffic flow problems)[2]

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[1]See A. Charnes and W.W. Cooper, "A Strategy for Making Models in Linear Programming," in R. Machol, et. al., eds., <u>Systems Engineering Handbook</u> (New York: McGraw-Hill, Inc., 1963).

[2]See Chapter XX in A. Charnes and W.W. Cooper, <u>Management Models and Industrial Applications of Linear Programming</u> (New York: John Wiley & Sons, Inc., 1961).

2. Operational Uses and Implementation Requirements--Ruggedness in the Face of Data Deficiencies and Computational Power (Example: transforming game theoretic formulation of traffic flow problem to a linear programming model to secure access to such ruggedness and computational power).

3. Interpretation--A use of extremal principles can facilitate and provide access to interpretations that might otherwise not be possible (Example: optimization of traffic flow example provided access to equilibrium characterizations which would not have been apparent from other approaches such as unguided simulation, etc.).

B. Flexibility and Extension
<u>Comment</u>: These properties provide yet another way to evaluate one or more extremal methods for a particular application. For instance this may be done by reference to whether they provide ready access to:

1. Alternative Representations and Results:

   a. Duality--For instance linear programming provides access to a dual problem and its solution at an optimum may be used to study the original problem.

   b. Non-Extremal Focus--For instance a recourse to an extremal model may be used to guide or control a simulation which, lacking such a guide, may be inefficient and ambiguous as well.

2. Alternative Methods or Mechanisms:

   a. Decentralization--Decomposition for both computation and control of decentralized operations by placing bounds or providing guides for latter.

   b. Feedback--Interactive and Iterative Methods which provide known improvements in wanted directions.

C. Ease of Explanation and Development

1. For Application and Managerial Use as in, say, a tradeoff analysis where assurance can be given that these are requisites (i.e., optimal) tradeoff terms.

2. For Guiding Further Scientific Research as when, say, the deeper analysis provided by an optimizing characterization ties together and reveals relations between many seemingly disparate problems or approaches to problems.

D. Entrance to New Methods or New Problems

3. Extensions and Further Developments

This last point, D., brings us back to the point at which the above analysis began but now this can be done from a new and better perspective perhaps. Recall that we said that we would have liked to have been able to arrange a two-way array with which to match problems and methods with related characterizations and evaluations. As is evident now, however, it is even better to regard this in terms of expandable or refinable depictions. Thus as new methods and especially methods involving extremal approaches are developed (e.g., from previously existing methods) then one might expect new problems to become open to attack. Conversely as one approaches new problems, or refinement of old ones, it may become evident where and in what forms new methods or combinations of preexisting methods are wanted.

An example from some of our own recent experiences with developing manpower planning models in conjunction with R. J. Niehaus and others at the U. S. Navy[3] may be used for illustration. In response to this problem the state of the modelling art was altered by, for the first time, including

[3]See A. Charnes, W. W. Cooper and R. J. Niehaus, Studies in Manpower Planning (Washington: U. S. Navy Office of Civilian Manpower Management, 1972).

Markovian elements in a goal-programming format. Next the customary divisions between "manpower planning" and "career analysis" were seen to be capable of simultaneous rather than separate treatment. This has led to a reconsideration of the ideas of organization design into a new dynamic format, and so on. Thus new and previously unknown "problem-possibilities" were opened for inclusion in our hypothetical table. The resulting problem enlargements then made it necessary to consider how it improves the computation power of goal programming if possible, and this has now been done[4] for the first time since its original invention.[5]

As should now be clear, the indicated enlargement of our matrix is not restricted only to categories as perceived or stated by logistics managers. This is not to denigrate in any way the importance of the latter as a source, however, but rather to emphasize that it is not the only source. New methods resulting from research can evidently open ways to uncovering and identifying previously unperceived problem possibilities and this, too, needs to be born in mind as we proceed to flesh out our discussion of extremal methods by reference to some of the history of these developments.

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[4]See A. Charnes, W. W. Cooper, D. Klingman, and R. J. Niehaus, "Convex Goal Programming."

[5]The ideas of goal programming were, curiously enough, first set forth in a problem that was also set in a personnel-manpower planning context. See A. Charnes, W. W. Cooper and R. O. Ferguson, "Optimal Estimation of Executive Compensation," Management Science I, No. 2, 1954.

4. The Survey

What follows is a compilation of articles that have appeared in the Naval Research Logistics Quarterly which have dealt with extremal principles applied to problems of logistics, considered in its most general sense.

This collection is by no means complete, and classifications are of necessity somewhat arbitrary, but it is hoped that these articles will give some examples of the development of extremal principles and their applications in logistics.

We have attempted to preface these classifications with a few introductory comments and include brief descriptions with the articles.

5. Linear Programming Methods

The development of efficient linear programming (LP) methods, paralleled by the rapid growth in computational capacity, has been central to the widespread application of mathematical management tools in the post war era.

With these developments, large linear programming models are increasingly amenable to solution. Models of a useful size have become practical as operational tools.

Beyond the benefits in linear programming applications, these methods have enabled solution of more general problems. Linear programs are repeatedly solved in some nonlinear, integer, and combinatorial algorithms, as well as some game theoretic solution procedures. Existence of efficient

LP codes is essential to the success of such approaches.

The availability of efficient LP procedures gives a direction of attack for solution of more complex problems. A problem that can be shown equivalent to, or approximable by a linear program, even a large such program, is accessible to solution.

Of the articles listed here, C. E. Lemke's "The Dual Method of Solving the Linear Programming Problem", NRLQ 1:1, pp. 36-47, March 1954, is of particular importance. Lemke's approach from the dual side was a fundamental contribution to linear programming methods. Elements of this approach have recurred frequently in dual and primal-dual algorithms.

[5.1] Lemke, C. E. "The Dual Method of Solving the Linear Programming Problem," NRLQ 1, 1 pp. 36-47 (May 1954).

[5.2] Alway, G. G. "A Triangularization Method for Computations in Linear Programming," NRLQ 9, 3 & 4 pp. 163-180 (September - December 1962).

[5.3] Beale, E. M. L. "Cycling in the Dual Simplex Algorithm," NRLQ 2, 4 pp. 269-275 (December 1955).

[5.4] Gass, S., and T. Saaty "The Computational Algorithm for the Parametric Objective Function," NRLQ 2, 1 & 2 pp. 39-45 (March - June 1955).

[5.5] Graves, G. W. "A Complete Constructive Algorithm for the General Mixed Linear Programming Problem," NRLQ 12, 1 pp. 1-34 (March 1965).

[5.6] Greenberg, H. "Note on a Modified Primal-Dual Algorithm to Speed Convergence in Solving Linear Programs," NRLQ 16, 2 pp. 271-273 (June 1969).

[5.7] Hadley, G. F. and M. A. Simmonard "A Simplified Two-Phase Technique for the Simplex Method," NRLQ 6, pp. 193-208 (September 1959).

[5.8] Harris, M. Y. "A Mutual Primal-Dual Linear Programming Algorithm," NRLQ 17, 2 pp. 199-206 (June 1970).

[5.9] Hartman, J. K. and L. S. Lasdon "A Generalized Upper Bounding Method for Doubly Coupled Linear Programs," NRLQ 17, 4 pp. 411-429 (December 1970).

[5.10] Jacobs, W. "Loss of Accuracy in Simplex Computations," NRLQ 4, 1 pp. 89-94 (March 1957).

[5.11] Kelley, J. E. Jr. "A Threshold Method for Linear Programming," NRLQ 4, 1 pp. 35-45 (March 57).

[5.12] Marshall, K. T. and J. W. Suurballe "A Note on Cycling in the Simplex Method," NRLQ 16, 1 pp. 121-137 (March 1969).

[5.13] Nemhauser, G. L. "Decomposition of Linear Programs by Dynamic Programming," NRLQ 11, 2 & 3 pp. 191-195 (June - September 1964).

[5.14] Thompson, P. M. "Editing Large Linear Programming Matrices," NRLQ 4, 1 pp. 95-100 (March 1957).

[5.15] Wagner, H. M. "The Dual Simplex Algorithm for Bounded Variables," NRLQ 5, 3 pp. 257-261 (September 1958).

6. Theory of Games

Military research and applications have been a major force in the development of the theory of games, subsequent to J. Von Neumann and O. Morgenstern's foundation of game theory in 1944.

A number of articles dealing with game theory have appeared in the Naval Research Logistics Quarterly. The diversity of the articles listed below is a small indication of the wide scope of possible application of the theory of games.

Articles in the NRLQ have dealt with formulation and characterization of specific problems, game theoretic approaches to classes of problems, development and extension of the theory of games, and solution procedures.

Examples of current relevance are D. Blackwell's "On Multi-Component Attrition Games," NRLQ 1, 3 pp. 210-216, September 1954, and L. S. Shapley's "Equilibrium Points in Games With Vector Payoffs," NRLQ 6, 1 pp. 57-61, March 1959. The theory of vector payoff games is still being developed and the concepts of these earlier articles retain relevance.

[6.1] Blackwell, D. "On Multi-Component Attrition Games" NRLQ 1, 3 pp. 210-216 (September 1954).

[6.2] Shapley, L. S. "Equilibrium Points in Games with Vector Payoffs", NRLQ 6, pp. 57-61 (March 1959).

[6.3] Agnew, R. A. and R. B. Hempley "Finite Statistical Games and Linear Programming," NRLQ 18, 1 pp. 99-102 (March 1971).

[6.4] Beale, E. M. L. and G. P. M. Heselden "An Approximation Method of Solving Blotto Games," NRLQ 9, 2 pp. 65-79 (June 1962).

[6.5] Blackett, D. W. "Pure Strategy Solutions of Blotto Games," NRLQ 5, 2 pp. 107-109 (June 1958).

[6.6] Blackett, D. W. "Some Blotto Games," NRLQ 1, 1 pp. 55- (March 1954).

[6.7] Braithwaite, R. B. "A Terminating Iterative Algorithm for Solving Certain Games and Related Sets of Linear Equations," NRLQ 6, pp. 63-74 (March 1959).

[6.8] Charnes, A. and R. G. Schroeder "On Some Stochastic Tactical Antisubmarine Games," NRLQ 14, 3 pp. 291-311 (September 1967).

[6.9] Chattopadhyay, R. "Differential Game Theoretic Analysis of a Problem of Warfare," NRLQ 16, 3 pp. 435-441 (September 1969).

[6.10] Cohen, N. D. "An Attack-Defense Game with Matrix Strategies," NRLQ 13, 4 pp. 391-402 (December 1966).

[6.11] Danskin, J. M. "A Game Over Spaces of Probability Distributions," NRLQ 11, 2 & 3 pp. 157-189 (June-September 1964).

[6.12] Danskin, J. M. "Fictitious Play for Continuous Games," NRLQ 1, 4 pp. 313-320 (December 1954).

[6.13] Davis, M. and M. Maschler "The Kernel of a Cooperative Game," NRLQ 12, 3 & 4 pp. 223-259 (September-December 1965).

[6.14] Eisenman, R. L. "Alliance Games of N-Persons," NRLQ 13, 4 pp. 403-411 (December 1966).

[6.15] Griesmer, J. H. and M. Shubik "Toward a Study of Bidding Processes · Some Constant-Sum Games," NRLQ 10, 1 pp. 11-21 (March 1963).

[6.16] Griesmer, J. H. and M. Shubik "Toward A Study of Bidding Processes, Part II.: Games with Capacity Limitations," NRLQ 10, 2 pp. 151-173 (June 1963).

[6.17] Griesmer, J. H. and M. Shubik "Toward A Study of Bidding Processes, Part III: Some Special Models," NRLQ 10, 3 pp. 199-217 (September 1963).

[6.18] Griesmer, J. H., R. E. Levitan, and M. Shubik "Towards A Study of Bidding Processes, Part IV; Games With Unknown Costs," NRLQ 14, 4 pp. 415-433 (December 1967).

[6.19] Hale, J. K. and H. H. Wicke "An Application of Game Theory to Special Weapons Evaluation," NRLQ 4, 4 pp. 347-356 (December 1957).

[6.20] Hershkowitz, M. "A Computational Note on VonNeumann's Algorithm for Determining Optimal Strategy," NRLQ 11, 1 pp. 75-78 (March 1964).

[6.21] Isbell, J. R. and W. H. Marlow "Attrition Games," NRLQ 3, 1-2 pp. 71-94 (March-June 1956).

[6.22] Marchi, E. "Simple Stability of General N-Person Games," NRLQ 14, 2 pp. 163-171 (June 1967).

[6.23] Maschler, M. "A Price Leadership Method for Solving the Inspector's Non-Constant-Sum Game," NRLQ 13, 1 pp. 11-33 (March 1966).

[6.24] Maschler, M. "The Inspector's Non-Constant-Sum Game: Its Dependence on a System of Detectors," NRLQ 14, 3 pp. 275-290 (September 1967).

[6.25] Moglewer, S. and C. Payne "A Game Theory Approach to Logistics Allocation," NRLQ 17, 1 pp. 87-97 (March 1970).

[6.26] Mond, B. "On The Direct Sum and Tensor Product of Matrix Games," NRLQ 11, 2 & 3 pp. 205-215 (June-September 1964).

[6.27] Morrill, J. E. "One-Person Games of Economic Survival," NRLQ 13, 1 pp. 49-69 (March 1966).

[6.28] Owen, G. "Political Game," NRLQ 18, 3 pp. 345-355 (September 1971).

[6.29] Peleg, B. "An Inductive Method for Constructing Minimal Balanced Collections of Finite Sets," NRLQ 12, 2 pp. 155-162 (June 1965).

[6.30] Peleg, B. "Utility Functions of Money for Clear Games," NRLQ 12, 1 pp. 57-63 (March 1965).

[6.31] Pruitt, W. E. "A Class of Dynamic Games," NRLQ 8, 1 pp. 55-78 (March 1961).

[6.32] Sakaguchi, M. "Pure Strategy Solutions to Blotto Games in Closed Auction Bidding," NRLQ 9, 3 & 4 pp. 253-263 (September-December 1962).

[6.33] Shapley, L. S. "On Balanced Sets and Cores," NRLQ 14, 4 pp. 453-460 (December 1967).

[6.34] Shubik, Martin and G. L. Thompson "Games of Economic Survival," NRLQ 6 pp. 111-125 (June 1959).

[6.35] Sweat, C. W. "Adaptive Competitive Decision in Repeated Play of a Matrix Game with Uncertain Entries," NRLQ 15, 3 pp. 425-448 (September 1968).

[6.36] Thompson, S. P. and A. J. Ziffer "The Watchdog and the Burglar," NRLQ 6 pp. 165-172 (June 1959).

[6.37] Thrall, R. M. and W. F. Lucas "N-Person Games in Partition Function Form," NRLQ 10, 4 pp. 281-298 (December 1963).

[6.38] Verhulst, M. "The Concept of a 'Mission'," NRLQ 3, 1-2 pp. 45-57 (March-June 1956).

[6.39] VonNeumann, J. "A Numerical Method to Determine Optimal Strategy," NRLQ 1, 2 pp. 109-115 (June 1954).

[6.40] Yasuda, Y. "A Note on the Core of a Cooperative Game Without Side Payment," NRLQ 17, 1 pp 143-149 (March 1970).

[6.41] Zachrisson, L. E. "A Tank Duel with Game-Theoretic Implications," NRLQ 4, 2 pp. 131-138 (June 1957).

7. Transportation and Assignment Problems

Transportation problems were among the first linear programs studied e.g. Kantorovich(1939),Hitchcock (1941), and Koopmans (1947). The classical Hitchcock-Koopmans transportation, or distribution, problem and the assignment problem are characterized by a special structure, but there are many other possible transportation problems.

Specialized algorithms for the distribution and assignment problems were developed at an early date, using this special structure to advantage for greater efficiency and the ability to handle larger problems than general codes. Examples are H. W. Kuhn's "The Hungarian Method for the Assignment Problem," NRLQ 2, 1 & 2 pp. 83-97, March-June 1955; A. Charnes and W. W. Cooper's "Stepping Stone" Method (1954), and G. B. Dantzig's "Row-Column Sum" Method (1951).

Articles in the NRLQ have dealt with algorithms for the distribution and assignment problems, extensions of these problems, and formulation of other problems of transportation.

An article of importance here is L. R. Ford and D. R. Fulkerson's "A Primal-Dual Algorithm for the Capacitated Hitchcock Problem," NRLQ 4, 1 pp. 47-54, March 1957. This article extended the Ford-Fulkerson network flow algorithm to the important case of the distribution problem where routes have limited capacities. Ford -Fulkerson methods, which rigorized Boldyreff's heuristic "method of flooding", have been a standard approach in distribution and network problems.

[7.1] Ford, L. R. Jr. and D. R. Fulkerson "A Primal Dual Algorithm for the Capacitated Hitchcock Problem," NRLQ 4, 1 pp. 47-54 (March 1957).

[7.2] Beale, E. M. L. "An Algorithm for Solving the Transportation Problem When the Shipping Cost Over Each Route Is Convex," NRLQ 6 pp. 43-56 (March 1959).

[7.3] Bellmore, M., W. D. Eklof, and G. L. Nemhauser "A Decomposable Transshipment Algorithm for a Multi-period Transportation Problem," NRLQ 16, 4 pp. 517-524 (December 1969).

[7.4] Briggs, F. E. A. "Solution of the Hitchcock Problem With One Single Row Capacity Constraint Per Row by the Ford-Fulkerson Method," NRLQ 9, 2 pp. 107-120 (June 1962).

[7.5] Charnes, A., F. Glover, and D. Klingman "The Lower Bounded and Partial Upper Bounded Distribution Model," NRLQ 18, 2 pp. 277-281 (June 1971).

[7.6] Gaddum, J. W., A. J. Hoffman, and D. Sokolowsky "On The Solution of the Caterer Problem," NRLQ 1, 3 pp. 223-229 (September 1954).

[7.7] Galler, B. A. and P. S. Dwyer "Translating the Method of Reduced Matrices to Machines," NRLQ 4, 1 pp. 55-71 (March 1957).

[7.8] Garfinkel, R. S. and M. R. Rao "The Bottleneck Transportation Problem," NRLQ 18, 4 pp. 465-472 (December 1971).

[7.9] Gassner, B. J. "Cycling in the Transportation Problem," NRLQ 11, 1 pp. 43-58 (March 1964).

[7.10] Glicksman, S., L. Johnson, and L. Eselson "Coding the Transportation Problem," NRLQ 7, 2 pp. 169-183 (June 1960).

[7.11] Hammer, P. L. "Communication on 'The Bottleneck Transportation Problem' and Some Remarks on the Time Transportation Problem," NRLQ 18, 4 pp. 487-490 (December 1971).

[7.12] Hammer, P. L. "Time Minimizing Transportation Problems," NRLQ 16, 3 pp. 345-357 (September 1969).

[7.13] Hoffman, A. J. and H. M. Markowitz "A Note on Shortest Path, Assignment, and Transportation Problems," NRLQ 10, 4 pp. 375-379 (December 1963).

[7.14] Holladay, J. "Some Transportation Problems and Techniques for Solving Them," NRLQ 11, 1 pp. 15-42 (March 1964).

[7.15] Jacobs, W. "The Caterer Problem," NRLQ 1, 2 pp. 154-165 (June 1954).

[7.16] Kuhn, H. W. "Variants of the Hungarian Method for Assignment Problems," NRLQ 3, 4 pp. 253-258 (December 1956).

[7.17] Kuhn, H. W. "The Hungarian Method for the Assignment Problem," NRLQ 2, 1 & 2 pp. 83-97 (March-June 1955).

[7.18] Lagemann, J.J. "A Method for Solving the Transportation Problem," NRLQ 14, 1 pp. 89-99 (March 1967).

[7.19] Prager, W. "Numerical Solution of the Generalized Transportation Problem," NRLQ 4, 3 pp. 253-261 (September 1957).

[7.20] Rigby, F. D. "An Analog and Derived Algorithm for the Dual Transportation Problem," NRLQ 9, 2 pp. 81-96 (June 1962).

[7.21] Simmonard, M. A. and G. F. Hadley "Maximum Number of Iterations in the Transportation Problem," NRLQ 6 pp. 125-129 (June 1959).

[7.22] Szwarc, W. "Some Remarks on the Time Transportation Problem," NRLQ 18, 4 pp. 473-485 (December 1971).

[7.23] Szwarc, W. "The Transportation Paradox," NRLQ 18, 2 pp. 185-202 (June 1971).

8. Integer Programming

An important special case of mathematical programming requires that elements, or all, of the solution be integer valued. Many problems to be modelled require allocation of indivisible units, such as men, ships, or aircraft, or include decision variables that can be represented as (0, 1) integer variables. Although distribution and assignment problems in integers produce integer solutions, this situation does not hold for more general cases. As it is known that continuous approximations to discrete problems can be distinctly suboptimal, exact integer solutions become of interest.

R. E. Gomory's cutting plane algorithm and solution of the integer programming problem, 1958, was a major development in the theory of integer linear programming and most subsequent work springs from this development. Some examples will be found below.

Articles in the NRLQ have dealt with alternative cutting plane approaches, solution characterization, and the central question of finite convergence.

An example is R. E. Gomory and A. J. Hoffman's "On The Convergence of an Integer Programming Process," NRLQ 10, 2 pp. 121-123, June 1963, which characterizes Dantzig cuts and shows the process not to converge in the general case. The Bowman-Nemhauser paper shows that Charnes and Cooper's modified cuts do give convergence.

[8.1] Gomory, R. E. and A. J. Hoffman "On the Convergence of an Integer Programming Process," NRLQ 10, 2 pp. 121-123 (June 1963).

[8.2] Bowman, V. J. Jr. and G. L. Nemhauser "A Finiteness Proof for Modified Dantzig Cuts in Integer Programming," NRLQ 17, 3 pp. 309-313 (September 1970).

[8.3] Dantzig, G. B. "Note on Solving Linear Programs in Integers," NRLQ 6 pp. 75-77 (March 1959).

[8.4] Robillard, P. "(0, 1) Hyperbolic Programming Problems," NRLQ 18, 1 pp. 47-57 (March 1971).

[8.5] Salkin, H. M. and P. Breining "Integer Points on the Gomory Fractional Cut (Hyperplane)," NRLQ 18, 4 pp. 491-496 (December 1971).

9. Economics

Problems of economics have been closely associated with the development of mathematical programming. Linear programming economic models and economic interpretations in mathematical programming were developed early in the history of linear programming.

Leontief's developments of input-output models of interindustry economics, 1936, was an important related development that served as one of the spurs to the development of linear programming. Input-output models were subsequently fused with mathematical programming models to produce powerful econometric models.

Articles in the NRLQ have dealt with Leontief models, linear and goal programming applications in economics, and characterization and extension of VonNeumann's economic equilibrium model. The Von Neumann model, 1937, is of current interest for its treatment of an expanding, rather than static, economy and its provision of growth index possibly superior to GNP (see the Morgenstern-Thompson paper). Another important article was D. Gale's "A Note on Global Instability of Competitive Equilibrium," NRLQ 10, 1 pp. 81-87, March 1963, which established a class of examples for which the competitive equilibrium is not globally stable.

[9.1] Gale, D. "A Note on Global Instability of Competitive Equilibrium," NRLQ 10, 1 pp. 81-87 (March 1963).

[9.2] Enzer, H., S. D. Berry, and J. I. Martin, Jr. "Economic Impact and the Notion of Compensated Procurement," NRLQ 15, 1 pp. 63-79 (March 1968).

[9.3] Enzer, H. and D. C. Dellinger "On Some Economic Concepts of Multiple Incentive Contracting," NRLQ 15, 4 pp. 477-489 (December 1968).

[9.4] Frisch, H. "Consumption, the Rate of Interest and the Rate of Growth in the Von Neumann Model," NRLQ 16, 4 pp. 459-484 (December 1969).

[9.5] Manne, A. S. "Comments on Interindustry Economics," NRLQ 7, 4 pp. 385-389 (December 1960).

[9.6] Morgenstern, O. and G. L. Thompson "An Open Expanding Economy Model," NRLQ 16, 4 pp. 443-457 (December 1969).

[9.7] Quandt, R. E. "Probabilistic Errors in the Leontief System," NRLQ 5, 2 pp. 155-170 (June 1958).

[9.8] Quandt, R. E. "On the Solution of Probabilistic Leontief Systems," NRLQ 6, 4 pp. 295-305 (December 1959).

[9.9] Spivey, W. A. and H. Tamura "Goal Programming in Econometrics," NRLQ 17, 2 pp. 183-192 (June 1970).

[9.10] Wong, Y. K. "An Elementary Treatment of an Input-Output System," NRLQ 1, 4 pp. 321-326 (December 1954).

[9.11] Wurtele, Z. S. "A Note on Pyramid Building," NRLQ 8, 4 pp. 377-379 (December 1961).

10. Quadratic and Quadratic Assignment Problems

Quadratic programming, optimization of a second degree objective function subject to linear constraints, was a natural extension of linear programming. The quadratic objective function is attractive as a second order approximation of more general functions and has a physical basis in terms of euclidean distance, area, power, etc.

Specialized algorithms were developed for this case because of its importance and relative accessibility. Articles in the NRLQ have dealt with algorithms and solution approaches to this problem and the special case of the quadratic assignment problem.

An important early algorithm was given by M. Frank and P. Wolfe "An Algorithm for Quadratic Programming," NRLQ 3, 1&2 pp. 95-109, March-June 1956. The Frank-Wolfe Algorithm solves a linear program at each iteration to determine step size along the gradient from the current point, relying for successful implementation upon efficient LP codes. Because of slow or non-convergence numerically sometimes, other algorithms by Beale, Van de Panne, Whinston and others have come to the fore.

[10.1] Frank, M. and P. Wolfe "An Algorithm for Quadratic Programming," NRLQ 3, 1-2 pp. 95-109 (March-June 1956).

[10.2] Beale, E. M. L. "On Quadratic Programming," NRLQ 6 pp. 227-243 (September 1959).

[10.3] Hildreth, C. "A Quadratic Programming Procedure," NRLQ 4, 1 pp. 79-85 (March 1957).

[10.4] Gaschutz, G. K. and J. H. Ahrens "Suboptimal Algorithms for the Quadratic Assignment Problem," NRLQ 15, 1 pp. 49-62 (March 1968).

[10.5] Greenberg, H. "A Quadratic Assignment Problem Without Column Constraints," NRLQ 16, 3 pp. 417-421 (September 1969).

[10.6] Markowitz, H. "The Optimization of a Quadratic Function Subject to Linear Constraints," NRLQ 3, 1-2 pp. 111-133 (March-June 1956).

[10.7] Pierce, J. F. and W. B. Crowston "Tree Search Algorithms for Quadratic Assignment Problems," NRLQ 18, 1 pp. 1-36 (March 1971).

[10.8] Van de Panne, C. and A. Whinston "Simplicial Methods for Quadratic Programming," NRLQ 11, 4 pp. 273-302 (December 1964).

[10.9] Whinston, A. "The Bounded Variable Problem--An Application of the Dual Method for Quadratic Programming," NRLQ 12, 2 pp. 173-179 (June 1965).

11. Probabilistic Programming

The problems we have been examining can be called deterministic in that parameters are assumed to be known constants. In many situations parameters are either not known precisely or are subject to variation, for example, future demand for goods or services.

There have been several approaches developed to deal with these problems. Among these are G. B. Dantzig's "Linear Programming Under Uncertainty," (1955) and A. Charnes, W. W. Cooper and G. H. Symond's "Chance Constrained Programming" (1954). Examples of both approaches will be found in the articles listed below.

One article of importance here is A. Charnes and W. W. Cooper's "Some Problems and Models for Time-Phased Transport Requirements, Chance Constrained Programs With Normal Deviates and Linear Decision

Rules," NRLQ 7, 4 pp. 533-544, December 1960, which further developed the theory of chance constrained programming in the context of investigating forward planning problems for time phased transport requirements.

[11.1] Charnes, A and W. W. Cooper "Some Problems and Models for Time-Phased Transport Requirements, Chance Constrained Programs with Normal Deviates and Linear Decision Rules," NRLQ 7, 4 pp. 533-544 (December 1960).

[11.2] Baron, D. P. "Information in Two-Stage Programming Under Uncertainty," NRLQ 18, 2 pp. 169-176 (June 1971).

[11.3] Bracken, J. and R. M. Soland "Statistical Decision Analysis of Stochastic Linear Programming Problems," NRLQ 13, 3 pp. 205-225 (September 1966).

[11.4] Midler, J. L. and R. D. Wollmer "Stochastic Programming Models for Scheduling Airlift Operations," NRLQ 16, 3 pp. 315-330 (September 1969).

[11.5] Yechiali, U. "A Note on a Stochastic Production-Maximizing Transportation Problem," NRLQ 18, 3 pp. 429-431 (September 1971).

12. Mathematical Programming Results

The NRLQ has served as a forum for a variety of articles describing theoretical results in mathematical programming. Topics have included constraint qualification, duality, and equivalent formulations in nonlinear and convex programming, semi-infinite programming, linear fractional programming, linear programming equivalences, and dynamic programming.

A major early article is S. Karlin's "The Structure of Dynamic Programming Models," NRLQ 2, 4 pp. 285-294, December 1955, but, the work of Arrow, Hurwicz and Uzawa, Bellman, Charnes and Cooper, and Shapley indicate the great scope and variety included.

[12.1] Karlin, S., "The Structure of Dynamic Programming Models," NRLQ 2, 4 pp. 285-294 (December 1955).

[12.2] Arrow, K. J., L. Hurwicz, and H. Uzawa "Constraint Qualifications in Maximization Problems," NRLQ 8, 2 pp. 175-191 (June 1961).

[12.3] Bellman, R. "Functional Equations and Successive Approximations in Linear and Nonlinear Programming," NRLQ 7, 1 pp. 63-83 (March 1960).

[12.4] Bellman, R. "Notes on the Theory of Dynamic Programming IV--Maximization over Discrete Sets," NRLQ 3, 1-2 pp. 67-70 (March-June 1956).

[12.5] Charnes, A. and W. W. Cooper "Nonlinear Network Flows and Convex Programming Over Incidence Matrices," NRLQ 5, 3 pp. 231-240 (September 1958).

[12.6] Charnes, A. and W. W. Cooper "Programming with Linear Fractional Functionals," NRLQ 9, 3&4 pp. 181-186 (September-December 1962).

[12.7] Charnes, A. and W. W. Cooper "Structural Sensitivity Analysis in Linear Programming and an Exact Product Form Left Inverse," NRLQ 15, 4 pp. 517-522 (December 1958).

[12.8] Charnes, A., W. W. Cooper, and K. O. Kortanek "On The Theory of Semi-Infinite Programming and a Generalization of the Kuhn-Tucker Saddle Point Theorem for Arbitrary Convex Functions," NRLQ 16, 1 pp. 41-51 (March 1969).

[12.9] Charnes, A., W. W. Cooper, and M. Miller "Dyadic Programs and Subdual Methods," NRLQ 8, 1 pp. 1-23 (March 1961).

[12.10] Charnes, A. and C. E. Lemke "Minimization of Non-Linear Separable Convex Functions," NRLQ 1, 4 pp. 301-312 (December 1954).

[12.11] Evans, J. P. "On Constraint Qualifications in Nonlinear Programming," NRLQ 17, 3 pp. 281-286 (September 1970).

[12.12] Gale, D. "The Basic Theorems of Real Linear Equations, Inequalities, Linear Programming and Game Theory," NRLQ 3, 3 pp. 193-200 (September 1956).

[12.13] Hoffman, A. J. "On Abstract Dual Linear Programs," NRLQ 10, 4 pp. 369-373 (December 1963).

[12.14] Joksch, H. C. "Programming With Fractional Linear Objective Functions," NRLQ 11, 2 & 3 pp. 197-204 (June-September 1964).

[12.15] Karush, William "A Theorem in Convex Programming," NRLQ 6 pp. 245-260 (September 1959).

[12.16] Malik, H. J. "A Note on Generalized Inverses," NRLQ 15, 4 pp. 605-612 (December 1958).

[12.17] Mangasarian, O. L. "Equivalence in Nonlinear Programming," NRLQ 10, 4 pp. 299-306 (December 1963).

[12.18] Martos, B. (Translated by A. & V. Whinston) "Hyperbolic Programming," NRLQ 11, 2&3 pp. 135-155 (June-September 1964).

[12.19] Saaty, T. L. "On Nonlinear Optimization In Integers," NRLQ 15, 1 pp. 1-22 (March 1968).

[12.20] Shapley, L. S. "Complements and Substitutes in the Optimal Assignment Problem," NRLQ 9, 1 pp. 45-48 (March 1962).

[12.21] Taylor, R. J. and S. P. Thompson "On a Certain Problem in Linear Programming," NRLQ 5, 2 pp. 171-187 (June 1958).

[12.22] Whinston, A. "Conjugate Functions and Dual Programs," NRLQ 12, 3&4 pp. 315-322 (September-December 1965).

[12.23] Zionts, S. "Programming With Linear Fractional Functionals," NRLQ 15, 3 pp. 449-451 (September 1968).

[12.24] Ritter, K. (Translated by M. Meyer) "A Method for Solving Nonlinear Maximum Problems Depending on Parameters," NRLQ 14, 2 pp. 147-162 (June 1967).

[12.25] D'Esopo, D. A. "A Convex Programming Procedure," NRLQ 6 pp. 33-43 (March 1959).

[12.26] Ericson, W. A. "On the Minimization of a Certain Convex Function Arising in Applied Decision Theory," NRLQ 15, 1 pp. 33-48 (March 1958).

[12.27] Zwart, P. B. "Nonlinear Programming the Choice of Direction by Gradient Projection," NRLQ 17, 4 pp. 431-438 (December 1970).

[12.28] Randolph, P. H. and G. E. Swinson "The Discrete Max-Min Problem," NRLQ 16, 3 pp. 309-314 (September 1969).

13. Fixed Charge Problems

Fixed charge problems are characterized by a fixed cost that is incurred any time an activity is operated at a nonzero level, in addition to variable costs. Minimum plant investments and set up charges are examples of this situation.

Articles in the NRLQ have dealt with formulations of the fixed charge and fixed charge transportation problems, and with algorithms for exact and approximate solutions.

W. M. Hirsch and G. B. Dantzig's "The Fixed Charge Problem," NRLQ 15, 3 pp. 413-424, September 1968, is a publication of the 1954 RAND paper which was among the first treatments of this problem and is of historical interest.

[13.1] Hirsch, W. M. and G. B. Dantzig "The Fixed Charge Problem," NRLQ 15, 3 pp. 413-424 (September 1968).

[13.2] Almogy, Y. and O. Levin "The Fractional Fixed Charge Problem," NRLQ 18, 3 pp. 307-315 (September 1971).

[13.3] Balinski, M. L. "Fixed Cost Transportation Problems," NRLQ 8, 1 pp. 41-54 (March 1961).

[13.4] Cooper, L. and C. Drebes "An Approximate Solution Method for the Fixed Charge Problem," NRLQ 14, 1 pp. 101-113 (March 1967).

[13.5] Denzler, D. R. "An Approximate Algorithm for the Fixed Charge Problem," NRLQ 16, 3 pp. 411-416 (September 1969).

[13.6] Dwyer, P. S. "Use of Completely Reduced Matrices in Solving Transportation Problems with Fixed Charges," NRLQ 13, 3 pp. 289-313 (September 1966).

[13.7] Kuhn, H. W. and W. J. Baumol "An Approximation Algorithm for the Fixed Charge Transportation Problem," NRLQ 9, 1 pp. 1-15 (March 1962).

[13.8] Steinberg, D. I. "The Fixed Charge Problem," NRLQ 17, 2 pp. 217-235 (June 1970).

14. Sequencing and Scheduling Problems

Sequencing and scheduling have been important areas of logistics research. These cover a wide range of practical problems, from scheduling of individual machines, to production scheduling, to project management.

Articles in the NRLQ indicate a part of this range and diversity. These articles have dealt with problem formulation and extensions, solution approaches and algorithms, and have included several reviews of the literature.

Solution approaches here include the use of linear programming, probabilistic programming, network methods, graph theory, dynamic programming, critical path methods, and simulation.

The first major paper was S. M. Johnson's "Optimal Two and Three Stage Production Schedules With Set Up Times Included," NRLQ 1, 1 pp. 61-68, March 1954, which presented the original results on multi-stage job shop scheduling. The Bartlett-Charnes cyclic scheduling results are still unique and in use in the railroads.

[14.1] Johnson, S. M. "Optimal Two and Three Stage Production Schedules With Set Up Times Included," NRLQ 1, 1 pp. 61-68 (March 1954).

[14.2] Arthanari, T. S. and A. C. Mukhopadhyay "A Note on a Paper by W. Szwarc," NRLQ 18, 1 pp. 135-138 (March 1971).

[14.3] Balas, E. "Machine Sequencing: Disjunctive Graphs and Degree-Constrained Subgraphs," NRLQ 17, 1 pp. 1-10 (March 1970).

[14.4] Bratley, P., M. Florian, and P. Robillard "Scheduling with Earliest Start and Due Date Constraints," NRLQ 18, 4 pp. 511-519 (December 1971).

[14.5] Burt, J. M. Jr, D. P. Gaver, and M. Perlas "Simple Stochastic Networks: Some Problems and Procedures," NRLQ 17, 4 pp. 439-459 (December 1970).

[14.6] Buzacott, J. A. and S. K. Dutta "Sequencing Many Jobs on a Multi-Purpose Facility," NRLQ 18, 1 pp. 75-82 (March 1971).

[14.7] Cremeans, J. E., R. A. Smith, and G. R. Tyndall "Optimal Multicommodity Network Flows with Resource Allocation," NRLQ 17, 3 pp. 269-279 (September 1970).

[14.8] Day, J. E. and M. P. Hottenstein "Review of Sequencing Research," NRLQ 17, 1 pp. 11-39 (March 1970).

[14.9] Elmaghraby, S. E. "The Machine Sequencing Problem--Review and Extensions," NRLQ 15, 2 pp. 205-232 (June 1968).

[14.10] Elmaghraby, S. E. "The Sequencing of 'Related' Jobs," NRLQ 15, 1 pp. 23-32 (March 1968).

[14.11] Evans, J. P. and F. J. Gould "Application of the GLM Technique to a Production Planning Problem," NRLQ 18, 1 pp. 59-74 (March 1971).

[14.12] Giffler, B. "Schedule Algebra: A Progress Report," NRLQ 15, 2 pp. 255-280 (June 1968).

[14.13] Giffler, B. "Scheduling General Production Systems Using Schedule Algebra," NRLQ 10, 3 pp. 237-255 (September 1963).

[14.14] Gleaves, V. B. "Cyclic Scheduling and Combinatorial Topology: Assignment and Routing of Motive Power to Meet Scheduling and Maintenance Requirements; Part I, A Statement of the Operation Problem of the Frisco Railroad," NRLQ 4, 3 pp. 203-205 (September 1957).

[14.15] Bartlett, T. E. and A. Charnes "Cyclic Scheduling and Combinatorial Topology: Assignment and Routing of Motive Power to Meet Scheduling and Maintenance Requirements; Part II, Generalization and Analysis," NRLQ 4, 3 pp. 207-220 (September 1957).

[14.16] Jackson, J. R. "An Extension of Johnson's Results on Job Lot Scheduling," NRLQ 3, 3 pp. 201-203 (September 1956).

[14.17] Klein, M. "Some Production Planning Problems," NRLQ 4, 4 pp. 269-286 (December 1957).

[14.18] Kortanek, K. O., D. Sodaro, and A. L. Soyster "Multi-Product Production Scheduling Via Extreme Point Properties of Linear Programming," NRLQ 15, 2 pp. 287-300 (June 1968).

[14.19] Levy, F. K., G. L. Thompson, and J. D. Wiest "Multiship, Multishop, Workload-Smoothing Program," NRLQ 9, 1 pp. 37-44 (March 1962).

[14.20] Maxwell, W. L. "The Scheduling of Economic Lot Sizes," NRLQ 11, 2 & 3 pp. 89-124 (June-September 1964).

[14.21] O'Neill, R. R. and J. K. Weinstock "The Cargo Handling System," NRLQ 1, 4 pp. 282-288 (December 1954).

[14.22] Smith, W. E. "Various Optimizers for Single Stage Production," NRLQ 3, 1-2 pp. 59-66 (March-June 1956).

[14.23] Spinner, A. H. "Sequencing Theory--Development to Date," NRLQ 15, 2 pp. 319-330 (March 1968).

[14.24] Srinivasan, V. "A Hybrid Algorithm for the One Machine Sequencing Problem to Minimize Total Tardiness," NRLQ 18, 3 pp. 317-327 (September 1971).

[14.25] Szwarc, W. "Elimination Methods in the m x n Sequencing Problem," NRLQ 18, 3 pp. 295-305 (September 1971).

[14.26] Szwarc, W. "On Some Sequencing Problems," NRLQ 15, 2 pp. 127-156 (June 1968).

[14.27] Te Chiang Hu and W. Prager "Network Analysis of Production Smoothing," NRLQ 6 pp. 17-24 (March 1959).

[14.28] Thompson, G. L. "Recent Developments in the Job-Shop Scheduling Problem," NRLQ 7, 4 pp. 585-589 (December 1960).

[14.29] Thompson, G. L. "CPM and DCPM Under Risk," NRLQ 15, 2 pp. 233-240 (June 1968).

[14.30] Von Lanzenauer, C. H. "Production and Employment Scheduling in Multistage Production Systems," NRLQ 17, 2 pp. 193-198 (June 1970).

[14.31] Wagner, Harvey M. "An Integer Programming Model for Machine Scheduling," NRLQ 6 pp. 131-140 (June 1959).

15. Mathematical Programming Applications

There have been a number of articles published in the NRLQ describing applications of mathematical programming models to problems in logistics, viewed in its most general sense. Some of these will be found in this section.

Categories of application have included, among others:

Aircraft procurement, deployment, operations, and rework (12, 13, 14, 22, 24, 27, 31, 53, 56)

The Naval tanker routing problem (2, 9, 19, 41)

Commercial and military shipping requirements and allocation (23, 36, 43)

The Naval electronics problem (46, 57, 59)

Transportation allocation and scheduling models (10, 17, 49, 51, 54, 58, 60)

Manpower planning (29, 40, 45, 52)

Bid evaluation (3, 28, 64)

Other financial and economic problems (8, 20, 33, 35, 55, 62, 63)

Supply system and resource allocation problems (1, 11, 30, 32, 37, 39)

Location problems (16, 21, 44, 47, 65)

A typical article of interest if J. Laderman, L. Gleiberman, and J. F. Egan's "Vessel Allocation by Linear Programming," (43) NRLQ 13, 3 pp. 315-320, September 1966, which describes the use of a linear programming model to schedule a heterogeneous fleet for a season's shipping requirements on the Great Lakes. Experience with this model in actual use is discussed. Bartlett's paper is of pathbreaking originality.

[15.1] Allen, S. G. "Redistribution of Total Stock Over Several User Locations," NRLQ 5, 4 pp. 337-345 (December 1958).

[15.2] Bartlett, T. E. "An Algorithm for the Minimum Number of Transport Units to Maintain a Fixed Schedule," NRLQ 4, 2 pp. 139-149 (June 1957).

[15.3] Beged-Dov, A. G. "Contract Award Analysis by Mathematical Programming," NRLQ 17, 3 pp. 297-307 (September 1970).

[15.4] Bellman, R. "Decision Making in the Face of Uncertainty--I," NRLQ 1, 3 pp. 230-232 (September 1954).

[15.5] Bellman, R. "Decision Making in the Face of Uncertainty-II," NRLQ 1, 4 pp. 327-332 (December 1954).

[15.6] Bellman, R. "Formulation of Recurrence Equations for Shuttle Process and Assembly Line," NRLQ 4, 4 pp. 321-334 (December 1957).

[15.7] Bellman, R. "On Some Applications of the Theory of Dynamic Programming," NRLQ 1, 2 pp. 141-153 (June 1954).

[15.8] Bellman, R. and S. Dreyfus "A Bottleneck Situation Involving Interdependent Industries," NRLQ 5, 4 pp. 307-314 (Decem-1958).

[15.9] Bellmore, M. "A Maximum Utility Solution to a Vehicle Constrained Tanker Scheduling Problem," NRLQ 15, 3 pp. 403-411 (September 1968).

[15.10] Bennington, G. and S. Lubore "Resource Allocation for Transportation," NRLQ 17, 4 pp. 471-484 (December 1970).

[15.11] Blitz, M. "Optimum Allocation of a Spares Budget," NRLQ 10, 2 pp. 175-191 (June 1963).

[15.12] Bracken, J. and P. R. Burnham "A Linear Programming Model for Minimum-Cost Procurement and Operation of Marine Corps Training Aircraft," NRLQ 15, 1 pp. 81-97 (March 1968).

[15.13] Bracken, J. and J. D. Longhill "Note on a Model for Minimizing Cost of Aerial Tankers Support of a Practice Bomber Mission," NRLQ 11, 4 pp. 359-364 (December 1964).

[15.14] Bracken, J. and K. W. Simmons "Minimizing Reductions in Readiness Caused by Time-Phased Decreases in Aircraft Overhaul and Repair Activities," NRLQ 13, 2 pp. 159-165 (June 1966).

[15.15] Bracken, J. and T. C. Varley "A Model for Determining Protection Levels for Equipment Classes Within a Set of Subsystems," NRLQ 10, 3 pp. 257-262 (September 1963).

[15.16] Breuer, M. A. "The Formulation of Some Allocation and Connection Problems As Integer Programs," NRLQ 13, 1 pp. 83-95 (March 1966).

[15.17] Charnes, A. and M. H. Miller "Mathematical Programming and Evaluation of Freight Shipment Systems, Part II--Analysis," NRLQ 4, 3 pp. 243-253 (September 1957).

[15.18] Cheney, L. K. "Linear Program Planning of Refinery Operations," NRLQ 4, 1 pp. 9-16 (March 1957).

[15.19] Dantzig, G. B. and D. R. Fulkerson "Minimizing the Number of Tankers to Meet a Fixed Schedule," NRLQ 1, 3 pp. 217-222 (September 1954).

[15.20] Daubin, S. C. "The Allocation of Development Funds: An Analytic Approach," NRLQ 5, 3 pp. 263-276 (September 1958).

[15.21] Davis, P. S. and T. L. Ray "A Branch-Bound Algorithm for the Capacitated Facilities Location Problem," NRLQ 16, 3 pp. 331-344 (September 1969).

[15.22] Dellinger, D. C. "An Application of Linear Programming to Contingency Planning: A Tactical Airlift Systems Analysis," NRLQ 18, 3 pp. 357-378 (September 1971).

[15.23] D'Esopo, D. A. and B. Lefkowitz "Note on An Integer Linear Programming Model for Determining a Minimum Embarkation Fleet," NRLQ 11, 1 pp. 79-82 (March 1964).

[15.24] Donis, J. N. and S. M. Pollock "Allocation of Resources to Randomly Occurring Opportunities," NRLQ 14, 4 pp. 513-527 (December 1967).

[15.25] Evans, G. W. "A Transportation and Production Model," NRLQ 5, 2 pp. 137-154 (June 1958).

[15.26] Finstman, S. I. "An Approximation Algorithm for an Optimum Aim-Points Algorithm," NRLQ 7, 2 pp. 151-167 (June 1960).

[15.27] Fitzpatrick, G. R., J. Bracken, M. J. O'Brien, L. G. Wentling, and J. C. Whiton "Programming the Procurement of Airlift and Sealift Forces: A Linear Programming Model for Analysis of the Least-Cost Mix of Strategic Deployment Systems," NRLQ 14, 2 pp. 241-255 (June 1967).

[15.28] Gainen, L., D. P. Honig, and E. D. Stanley "Linear Programming in Bid Evaluation," NRLQ 1, 1 pp. 49-54 (March 1954).

[15.29] Gass, S. I. "On the Distribution of Manhours to Meet Scheduled Requirements," NRLQ 4, 1 pp. 17-25 (March 1957).

[15.30] Gilbert, J. C. "A Method of Resource Allocation Using Demand Preference," NRLQ 11, 2&3 pp. 217-225 (June-September 1964).

[15.31] Gross, D. and R. M. Soland "A Branch and Bound Algorithm for Allocation Problems in Which Constraint Coefficients Depend upon Decision Variables," NRLQ 16, 2 pp. 157-174 (June 1969).

[15.32] Hadley, G. and T. M. Whitin "A Model for Procurement, Allocation, and Redistribution for Low Demand Items," NRLQ 8, 4 pp. 395-414 (December 1961).

[15.33] Hitchcock, D. F. and J. B. MacQueen "On Computing the Expected Discounted Return in a Markov Chain," NRLQ 17, 2 pp. 237-241 (June 1970).

[15.34] Houston, B. F. and R. A. Huffman "A Technique Which Combines Modified Pattern Search Methods with Composite Designs and Polynomial Constraints to Solve Constrained Optimization Problems," NRLQ 18, 1 pp. 91-98 (March 1971).

[15.35] Howard, G. T. and G. L. Nemhauser "Optimal Capacity Expansion," NRLQ 15, 4 pp. 535-550 (December 1968).

[15.36] Hunt, R. B. and E. F. Rosholdt "Determining Merchant Shipping Requirements in Integrated Military Planning," NRLQ 7, 4 pp. 545-575 (December 1960).

[15.37] Jewell, W. S. "Warehousing and Distribution of a Seasonal Product," NRLQ 4, 1 pp. 29-34 (March 1957).

[15.38] Karlin, S., W. E. Pruitt, and W. G. Madow "On Choosing Combinations of Weapons," NRLQ 10, 2 pp. 95-119 (June 1963).

[15.39] Karreman, H. F. "Programming the Supply of a Strategic Material--Part I, A Nonstochastic Model," NRLQ 7, 3 pp. 261-279 (September 1960).

[15.40] Karush, W. and A. Vazsonyi "Mathematical Programming and Employment Scheduling," NRLQ 4, 4 pp. 297-320 (December 1957).

[15.41] Kelley, J. E. "A Dynamic Transportation Model," NRLQ 2, 3 pp. 175-180 (September 1955).

[15.42] Kolesar, P. J. "Linear Programming and the Reliability of Multicomponent Systems," NRLQ 14, 3 pp. 317-327 (September 1967).

[15.43] Laderman, J., L. Gleiberman, and J. F. Egan "Vessel Allocation by Linear Programming," NRLQ 13, 3 pp. 315-320 (September 1966).

[15.44] Love, R. F. "Locating Facilities in Three-Dimensional Space by Convex Programming," NRLQ 16, 4 pp. 503-516 (December 1969).

[15.45] McCloskey, J. F. and F. Hanssmann "An Analysis of Stewardess Requirements and Scheduling for a Major Airline," NRLQ 4, 3 pp. 183-191 (September 1957).

[15.46] McShane, R. E. and Solomon, H. "Letter Concerning Naval Electronics Problem," NRLQ 5, 4 pp. 363-367 (December 1958).

[15.47] Nair, K. P. K. and R. Chandrasekaran "Optimal Location of a Single Service Center of Certain Types," NRLQ 18, 4 pp. 503-510 (December 1971).

[15.48] Noble, S. B. "Some Flow Models of Production Constraints," NRLQ 7, 4 pp. 401-419 (December 1960).

[15.49] O'Neill, R. R. "Scheduling of Cargo Containers," NRLQ 7, 4 pp. 577-584 (December 1960).

[15.50] Pollack, M. "Some Studies on Shuttle and Assembly-Line Processes," NRLQ 5, 2 pp. 125-136 (June 1958).

[15.51] Pruzan, P. M. and J. T. R. Jackson "The Many-Product Cargo Loading Problem," NRLQ 14, 3 pp. 381-390 (September 1967).

[15.52] Rau, J. G. "A Model for Manpower Productivity During Organization Growth," NRLQ 18, 4 pp. 543-559 (December 1971).

[15.53] Rice, E. W., J. Bracken, and A. W. Pennington "Allocation of Carrier-Based Aircraft Using Non-Linear Programming," NRLQ 18, 3 pp. 379-393 (September 1971).

[15.54] Saposnik, R., V. L. Smith, and A. R. Lindeman "Allocation of a Resource to Alternative Probabilistic Demands: Transport-Equipment Pool Assignments," NRLQ 6, pp. 193-208 (September 1959).

[15.55] Scherer, F. M. "Time-Cost Tradeoffs in Uncertain Empirical Research Projects," NRLQ 13, 1 pp. 71-82 (March 1966).

[15.56] Schwartz, A. N., J. A. Sheler, and C. R. Cooper "Dynamic Programming Approach to the Optimization of Naval Aircraft Rework and Replacement Policies," NRLQ 18, 3 pp. 395-414 (September 1971).

[15.57] Smith, J. W. "A Plan to Allocate and Procure Electronics Sets by the Use of Linear Programming Techniques and Analytical Methods of Assigning Values to Qualitative Factors," NRLQ 3, 3 pp. 151-162 (September 1956).

[15.58] Soyster, H. R. "Mathematical Programming and Evaluations of Freight Shipment Systems, Part I--Applications" NRLQ 4, 3 pp. 237-242 (September 1957).

[15.59] Suzuki, G. "Procurement and Allocation of Naval Electronic Equipments," NRLQ 4, pp. 1-7 (March 1957).

[15.60] Szwarc, W. "The Truck Assignment Problem," NRLQ 14, 4 pp. 529-557 (December 1967).

[15.61] Vergin, R. C. "Optimal Renewal Policies for Complex Systems," NRLQ 15, 4 pp. 523-534 (December 1958).

[15.62] Wagner, H. M. "A Postscript to 'Dynamic Problems in the Theory of the Firm'," NRLQ 7, 1 pp. 7-12 (March 1960).

[15.63] Wagner, H. M. and T. M. Whitin "Dynamic Problems in the Theory of the Firm," NRLQ 5, 1 p. 53 (March 1958).

[15.64] Waggener, H. A. and G. Suzuki "Bid Evaluation for Procurement of Aviation Fuel at DFSC: Case History," NRLQ 14, 1 pp. 115-129 (March 1967).

[15.65] Wesolowsky, G. O. and R. F. Love "Location of Facilities With Rectangular Distances Among Point and Area Destinations," NRLQ 18, 1 pp. 83-90 (March 1971).

[15.66] Whiton, J. C. "Some Constraints on Shipping in Linear Programming Models," NRLQ 14, 2 pp. 257-260 (June 1967).

16. Network Theory

Network theory has important application in logistics. Transportation and transshipment problems are but two examples of cases where network models and techniques have been used to advantage here. The Ford-Fulkerson Network flow approach to distribution problems was previously noted.

Articles in the NRLQ have treatments of the convex cost minimum flow network problem, interdiction and isolation problems with military interpretations, and network flow functions

One important early article in this field was G. B. Dantzig and D. R. Fulkerson's "Computation of Maximal Flows in Networks," NRLQ 2, 4 pp. 277-283 (December 1955). Shapley's incisive "Network Flow Functions" paper is also represented.

[16.1] Dantzig, G. B. and D. R. Fulkerson "Computation of Maximal Flows in Networks," NRLQ 2, 4 pp. 277-283 (December 1955).

[16.2] Bellmore, M. G. Bennington and S. Lubore "A Network Isolation Algorithm," NRLQ 17, 4 pp. 461-469 (December 1970).

[16.3] Ghare, P. M., D. C. Montgomery and W. C. Turner "Optimal Interdiction Policy for a Flow Network," NRLQ 18, 1 pp. 37-45 (March 1971).

[16.4] Glover, F. "Maximum Matching in a Convex Bipartite Graph," NRLQ 14, 3 pp. 313-316 (September 1967).

[16.5] Hu, T. C. "Minimum-Cost Flows in Convex-Cost Networks," NRLQ 13, 1 pp. 1-9 (March 1966).

[16.6] Jarvis, J. J. "On the Equivalence Between the Node-Arc and Arc-Chain Formulations for the Multi-Commodity Maximal Flow Problem," NRLQ 16, 4 pp. 525-529 (December 1959).

[16.7] Lubore, S. H., H. D. Ratliff and G. T. Sicilia "Determining the Most Vital Link in a Flow Network," NRLQ 18, 4 pp. 497-502 (December 1971).

[16.8] McMasters, A. W. and T. M. Mustin "Optimal Interdiction of a Supply Network," NRLQ 17, 3 pp. 261-268 (September 1970).

[16.9] Menon, V. V. "The Minimal Cost Flow Problem With Convex Costs," NRLQ 12, 2 pp. 163-172 (June 1965).

[16.10] Shapley, L. S. "On Network Flow Functions," NRLQ 8, 2 pp. 151-158 (June 1961).

[16.11] Wollmer, R. D. "Interception in a Network," NRLQ 17, 2 pp. 207-216 (June 1970).

17. Miscellaneous

Included here is a selection of articles from the NRLQ related to a variety of logistical or mathematical issues, not elsewhere classified.

An article of interest is R. R. O'Neill's "Analysis and Monte Carlo Simulation of Cargo Handling," NRLQ 4, 3 pp. 223-242 (September 1957), which presents an analysis of cyclic linked cargo handling systems and demonstrates use of simulation techniques in the solution of several examples.

[17.1] O'Neill, R. R. "Analysis and Monte Carlo Simulation of Cargo Handling," NRLQ 4, 3 pp. 223-242 (September 1957).

[17.2] Aumann, R. J. and J. B. Kruskal "The Coefficients in an Allocation Problem," NRLQ 5, 2 pp. 111-123 (June 1958).

[17.3] Aumann, R. J. and J. B. Kruskal "Assigning Quantitative Values to Qualitative Factors in the Naval Electronics Problem," NRLQ 6, pp. 1-16 (March 1959).

[17.4] Mellon, W. G. "A Selected, Descriptive Bibliography of References on Priority Systems and Related, Nonprice Allocators," NRLQ 5, 1 p. 17 (March 1958).

[17.5] Antosiewicz, H. A. "Analytic Study of War Games," NRLQ 2, 3 pp. 181-208 (September 1955).

[17.6] Flood, M. M. "Operations Research and Logistics," NRLQ 5, 4 pp. 323-335 (December 1958).

[17.7] Harary, F. and M. Richardson "A Matrix Algorithm for Solutions and r-Bases of a Finite Irreflexive Relation," NRLQ 6, 4 pp. 307-314 (December 1959).

[17.8] Pfanzagl, J. "A General Theory of Measurement-Applications to Utility," NRLQ 6, 4 pp. 283-294 (December 1959).

[17.9] Isaacs, R. "The Problem of Aiming and Evasion," NRLQ 2, 1 & 2 pp. 47-67 (March--June 1955).

[17.10] D'Esopo, D. A., H. L. Dixon and B. Lefkowitz "A Model for Simulating an Air-Transportation System," NRLQ 7, 3 pp. 213-220 (September 1960).

[17.11] Jackson, J. R. "Simulation Research on Job Shop Production," NRLQ 4, 4 pp. 287-295 (December 1957).

[17.12] Salveson, M. E. "Principles of Dynamic Weapon Systems Programming," NRLQ 8, 1 pp. 79-110 (March 1961).

[17.13] Brandenburg, R. G. and A. C. Stedry "Toward a Multi-Stage Information Conversion Model of the Research and Development Process," NRLQ 13, 2 pp. 129-146 (June 1966).

[17.14] Gouarary, M. H. "A Simple Rule for the Consolidation of Allowance Lists," NRLQ 5, 1 pp. 1-15 (March 1958).

[17.15] Hershkowitz, M. and S. B. Noble "Finding the Inverse and Connections of a Type of Large Sparse Matrix," NRLQ 12, 1 pp. 119-132 (March 1965).

[17.16] Wortham, A. W. and E. B. Wilson "A Backward Recursive Technique For Optimal Sequential Sampling Plans," NRLQ 18, 2 pp. 203-213 (June 1971).

18. Search and Surveillance Problems

There have been several articles in the NRLQ dealing with problems of search and surveillance. Some of those articles which employed extremal principles are listed here.

One example of these articles is C. Derman and M. Klein's "Surveillance of Multi-Component Systems: A Stochastic Travelling Salesman's Problem," NRLQ 13, 2 pp. 103-111, June 1966. Linear programming is used here to find the optimal sequence of inspections.

[18.1] Derman, C. and M. Klein "Surveillance of Multi-Component Systems: A Stochastic Travelling Salesman's Problem," NRLQ 13, 2 pp. 103-111 (June 1966).

[18.2] Derman, C. "On Minimax Surveillance Schedules" NRLQ 8, 4 pp. 415-419 (December 1961).

[18.3] Roeloffs, R. "Minimax Surveillance Schedules with Partial Information," NRLQ 10, 4 pp. 307-322.

[18.4] Roeloffs, R. "Minimax Surveillance Schedules for Replaceable Units," NRLQ 14, 4 pp. 461-471 (December 1967).

[18.5] Blachman, N. M. "Prolegomena to Optimum Discrete Search Procedures," NRLQ 6, 4 pp. 273-282 (December 1959).

[18.6] Dobbie, J. M. "Search Theory: A Sequential Approach," NRLQ 10, 4 pp. 323-334 (December 1963).

[18.7] Enslow, P. H. "A Bibliography of Search Theory and Reconnaissance Theory Literature," NRLQ 13, 2 pp. 177-202 (June 1966).

[18.8] Isbell, J. R. "An Optimal Search Pattern" NRLQ 4 4 pp. 357-359 (December 1957).

[18.9] Smith, M. W. and J. E. Walsh "Optimum Sequential Search With Discrete Locations and Random Acceptance Ratios," NRLQ 18, 2 pp. 159-167 (June 1971).